Optimizing the Throughput of Hazardous Waste Incinerators

This work presents a linear, steady-state model of a rotary kiln hazardous waste incinerator. Mass balance constraints, energy balance constraints, constraints typically imposed by incinerator permits, and constraints imposed by design limitations are all included in the model. The model is used to perform optimization studies, which maximize a measure of the incinerator's throughput. The sensitivity of the model to uncertainties in the chemical property values of the waste streams is also examined. For the spectrum of case studies examined, the optimal operating conditions are most sensitive to halogen content and water content of the wastes and are less sensitive to variations in heating value and nitrogen content.

Nasrin Behmanesh
Vasilios Manousiouthakis
David T. Allen
Department of Chemical Engineering
University of California
Los Angeles, CA 90024

Introduction

Incineration is an important component of the hazardous waste management policy of the United States. There are approximately 200 hazardous waste incinerators in this country (EPA, 1989), which destroy about 2 million tons per year of hazardous waste. Most of the incinerators are on-site facilities, responsible for destroying a well-defined set of wastes. These incinerators must be robust enough to handle variations in waste composition. Since the number of waste streams is limited, composition shifts can be dramatic if a single waste stream is changed. In contrast, the sixteen permitted commercial facilities (EPA, 1989) receive a wide variety of wastes from many generators. The wastes can vary in physical state (solid, liquid, sludge), chemical composition, and thermodynamic properties. They may be fed to the incinerator in continuous or batch mode. The goal of the commercial incinerator is to process the mix of wastes that generates the most net revenue.

For both the commercial and on-site incinerators, there are a large number of operational constraints. These constraints are imposed by the permit, by the design of the incinerator, and by operating practices. The operator of the incinerator must blend the wastes or control the waste flow rates, air flow rates, water spray flow rates, and auxiliary fuel flow rates such that all the constraints are satisfied. At the same time, the operator seeks to maximize the waste throughput or minimize the auxiliary fuel flow to the incinerator. Incinerator operators currently attempt to achieve these objectives using intuition and experience. The goal of this work is to present a systematic, optimization-based methodology that can address these objectives. The optimization methodology must account for the fact that waste stream properties are uncertain. In commercial incinerator operation, the uncertainty may be due to heterogeneous waste mixtures that cannot be sampled accurately or to inaccurate property measurements. For on-site incinerators, waste variability may be due to variations in the performance of the processes generating the wastes. In either case, incorporating measurement uncertainties into the incinerator model is essential.

The following sections present the model development, the solution procedure for the optimization problem, and a number of case studies. The case studies indicate that optimal operating conditions can be sensitive functions of the feed stream characteristics.

Incinerator Model Development

In this work, both the commercial and on-site incinerators will be assumed to be rotary kilns followed by an afterburner, as shown in Figure 1. The kiln performs the bulk of the combustion and the afterburner is used to insure complete waste destruction. In the generic configuration of Figure 1, the feed system consists of five sources of waste: one high heating value waste tank, one medium heating value waste tank, one low heating value waste tank, a sludge waste tank, and one direct burn injection port. Direct burn waste and sludge waste can only be fed to the kiln, while other types of wastes can be fed to both the kiln and the afterburner. The waste in each tank is characterized by ultimate analysis, which provides the weight percent of the elements C, H, O, S, P, Cl, F, Br, I, and N, along with H_2O and ash content.

Correspondence concerning this paper should be addressed to D. T. Allen.

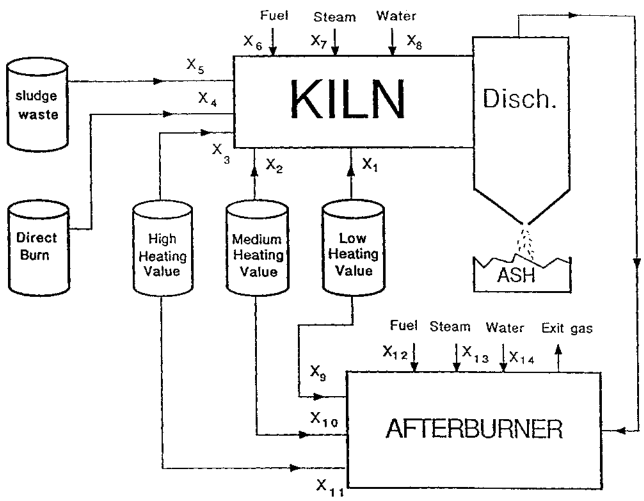

Figure 1. Incinerator configuration used in the model.

The heating value and density of the wastes are also known, to some level of accuracy. In addition to the waste streams, auxiliary fuel and atomizing water are occasionally fed to the incinerator.

The constraints that limit both commercial and on-site incinerator operation can be loosely grouped into those imposed by the design and those imposed by the permit. Typical design constraints include pump capacities and maximum temperature ratings. The design constraints used in this work were based on a piping and instrumentation diagram for a proposed incinerator. The constraints imposed by permits typically include minimum residence time and minimum temperatures. The permit constraints used in this work were taken from a proposed incinerator permit.

Thirty-four constraints, listed in Table 1, along with basic heat and material balance relationships, form the basis of the incinerator model. To make the model amenable to optimization calculations, a few of the constraints have been linearized, however, most of the constraints are inherently linear. As an example of a linear constraint, consider a constraint imposed by the permit, which limits the total amount of heat released in the kiln (constraint 25, Table 1). The constraint can be written as:

$$\sum_{i=1}^{6} \mathrm{HV}_i x_i \leq \mathrm{KHIN} \tag{1}$$

where HV_i and x_i are the heating values and flow rates of the six streams (high, medium, and low heating value wastes, direct burn, sludge, and auxiliary fuel) that can be fed to the kiln, and KHIN is the maximum allowed heat input. This heat release limitation serves two functions. It limits total waste throughput and it reduces the likelihood of hot spots that could damage incinerator components.

Most of the constraints used in this work can, in an analogous manner, be cast in linear form. The only constraints that are not inherently linear functions of waste stream flow rates are those involving temperature. These contraints have been linearized. Consider as an example a constraint on the minimum kiln operating temperature (constraint 27, Table 1) which is imposed by the permit to help insure complete waste destruction. In the full heat and mass balance for the incinerator, the temperature is calculated iteratively using Eq. 2:

$$\frac{\text{Heat release} - \text{heat loss}}{\sum_i \sum_g C_g \nu_{gi} x_i + C_p^{EA} \cdot \mathrm{EA}} = T_f - T_o \tag{2}$$

where T_f is the operating temperature, T_o is the ambient (reference) temperature, C_g is the molar heat capacity of combustion gas g, ν_{gi} is the number of moles of combustion gas g generated per unit mass of feed stream i, x_i is the mass flow rate of stream i, and C_p^{EA} and EA are the heat capacity and the flow rate of the excess air, respectively. In general, Eq. 2 is not linear in waste stream flow rates since the average heat capacity and, to a lesser extent, the heat loss term depend on waste stream flow rates. Casting a temperature constraint as a linear function of waste stream flow rates therefore requires some assumptions. We use the assumption of a constant heat loss for the kiln, KHLOS, and employ an average value of the heat capacity between the feed temperature and the minimum allowed operating temperature, C_g^*. These simplifications are justified if the incinerator operates close to its minimum allowed temperature. Since low heating value wastes often generate the most revenue and since the use of auxiliary fuel is generally minimized, many incinerators do operate at close to the minimum allowed temperature. A final assumption made in the analysis is that combustion is complete. Using these assumptions, the kiln temperature constraint can be cast in the linear form

$$\frac{\sum_i \mathrm{HV}_i x_i - \mathrm{KHLOS}}{\sum_i \sum_g C_g^* \nu_{gi} x_i + C_p^{EA} \mathrm{EA}} \geq T_f - T_o \tag{3}$$

Thus all the constraints used in this work are either linear or linearized. The inequalities representing all of the constraints are given in Table 1. Details of their derivation are given elsewhere (Behmanesh, 1990).

Formulation of the optimization problem

For many incinerators, the goal of the optimization is to maximize waste throughput or revenue. Both of these objectives are linear functions of the feed stream flow rates, with the following form:

$$C^T x = \sum_{\text{kiln}} C_i x_i + \sum_{\text{afterburner}} C_i x_i \tag{4}$$

where x is the vector of feed stream flow rates, and the C_i are weighting factors. These weighting factors are all unity if throughput is the objective function and they are equal to the incineration price if the objective function is revenue. In this work, a measure of throughput will be used as the objective function that is to be maximized. This function is simply waste throughput minus auxiliary fuel flow rate, as shown in Eq. 5.

$$C^T x = \left(\sum_{\text{kiln}} x_i + \sum_{\text{afterburner}} x_i \right) - x_{\text{fuel,kiln}} - x_{\text{fuel,afterburner}} \tag{5}$$

Equation 5 is the optimization function to be used throughout this work. It is to be maximized subject to the linear constraints given in Table 1. These constraint equations assume that all waste properties are accurate and precise. This is clearly not the

Table 1. Constraint Equations

Constraints 1 through 17 are imposed by the design of the incinerator.

1. Total feed rate of combined waste not exceeding CWAST:

$$\sum_{i=1}^{5} x_i + \sum_{j=9}^{11} x_j \le \text{CWAST}$$

2–9. Flow rates of waste streams within specified range:

$$\text{LX}_i \le x_i \le \text{UX}_i, \quad i = 1\text{–}5, 9\text{–}11$$

10–12. Tank combined flow rates limited by max. value:

$$x_1 + x_9 \le \text{MXL}$$
$$x_2 + x_{10} \le \text{MXM}$$
$$x_3 + x_{11} \le \text{MXH}$$

13–17. Vol. of waste in tanks maintained above min. value:

$$\text{VL}_T - \text{TVL} \ge [(x_1 + x_9) \cdot \text{RTB}]/\rho_1$$
$$\text{VM}_T - \text{TVM} \ge [(x_2 + x_{10}) \cdot \text{RTB}]/\rho_2$$
$$\text{VH}_T - \text{TVH} \ge [(x_3 + x_{11}) \cdot \text{RTB}]/\rho_3$$
$$\text{VDB}_T - \text{TVDB} \ge (x_4 \cdot \text{RTB})/\rho_4$$
$$\text{VSL}_T - \text{TVSL} \ge (x_5 \cdot \text{RTB})/\rho_5$$

Constraints 18 through 34 are imposed by the permit issued for the incinerator.

18. Max. kiln ash content:

$$\frac{\sum_{i=1}^{5} (\%\,\text{ash})_i x_i}{\sum_{i=1}^{5} x_i} \le \text{KASH}$$

or

$$\sum_{i=1}^{5} [(\%\,\text{ash})_i - \text{KASH}] \cdot x_i \le 0$$

19. Max. afterburner ash content:

$$\sum_{j=9}^{11} [(\%\,\text{ash})_j - \text{AASH}] \cdot x_j \le 0$$

20. Max. combined waste ash content:

$$\sum_{i=1}^{5} \{[(\%\,\text{ash})_i - \text{CASH}] \cdot x_i + \sum_{j=9}^{11} [(\%\,\text{ash})_j - \text{CASH}] \cdot x_j\} \le 0$$

21. Max. water content of combined waste:

$$\sum_{i=1}^{5} \{[(\%\,H_2O)_i - \text{CWATR}] \cdot x_i\} + \sum_{j=9}^{11} \{[(\%\,H_2O)_j - \text{CWATR}] \cdot x_j\} \le 0$$

22. Max. chlorine content of combined waste:

$$\sum_{i=1}^{5} \{[(\%\,\text{Cl})_i - \text{CHLOR}] \cdot x_i\} + \sum_{j=9}^{11} \{[(\%\,\text{Cl})_j - \text{CHLOR}] \cdot x_j\} \le 0$$

23. Max. sulfur content of combined waste:

$$\sum_{i=1}^{5} \{[(\%\,\text{S})_i - \text{CSULF}] \cdot x_i\} + \sum_{j=9}^{11} \{[(\%\,\text{S})_j - \text{CSULF}] \cdot x_j\} \le 0$$

24. Max. fuel nitrogen content of combined waste:

$$\sum_{i=1}^{5} \{[(\%\,\text{N})_i - \text{CNFUL}] \cdot x_i\} + \sum_{j=9}^{11} \{[(\%\,\text{N})_j - \text{CNFUL}] \cdot x_j\} \le 0$$

25. Max. kiln heat input:

$$\sum_{i=1}^{6} \text{HV}_i x_i \le \text{KHIN}$$

26. Max. total heat input to combined system of kiln and afterburner:

$$\sum_{i=1}^{6} \text{HV}_i x_i + \sum_{j=9}^{12} \text{HV}_j x_j \le \text{CHIN}$$

27. Min. temperature for the kiln:

$$\frac{\left(\sum_{i=1}^{8} \text{HV}_i x_i\right) - \text{KHLOS}}{\left(\sum_{i=1}^{8} \sum_{g=1}^{10} C_g^* \nu_{gi} x_i\right) + \text{KCPEA}} \ge (\text{KTEMP} - T_o)$$

28. Min. temperature for the afterburner:

$$\frac{\sum_{i=1}^{8} \text{HV}_i x_i + \sum_{j=9}^{14} \text{HV}_j x_j - \text{KHLOS} - \text{AHLOS}}{\sum_{i=1}^{8} \sum_{g=1}^{10} C_g^* \nu_{gi} x_i + \sum_{j=9}^{14} \sum_{g=1}^{10} C_g^* \nu_{gj} x_j + \text{ACPEA}} \ge (\text{ATEMP} - T_o)$$

29. Min. oxygen content of kiln exit gas:
Exit gas molar rate (mol/h):

$$\text{KEXIT} = \text{KCG} + (O_2)_{EA} + (N_2)_{EA}$$
$$= \text{KCG} + (1 + 79/21) \cdot \text{KEAP} \cdot (O_2)_{\text{stoich,K}}$$

Constraint:

$$(1 + \text{KEAP}) \cdot (O_2)_{\text{stoich,K}} - (O_2)_{\text{stoich,K}} \ge \text{KOXY} \cdot \text{KEXIT}$$

30. Min. oxygen content of afterburner exit gas:

$$\text{AEAP} \cdot (O_2)_{\text{stoich,A}} \ge \text{AOXY} \cdot \text{AEXIT}$$

31. Afterburner combustion gas velocity in the range of LAGV m/s and UAGV m/s:

$$\text{AEXIT} = \text{ACG} + (O_2)_{EA} + (N_2)_{EA}$$
$$= \text{ACG} + (1 + 79/21) \cdot \text{AEAP} \cdot (O_2)_{\text{stoich,A}}$$

Specific gas velocity (m/s)

$$\text{AGV} = \text{AGAV}/\text{AA}$$

Constraint:

$$\text{LAGV} \le \text{AGV} \le \text{UAGV}$$

32. Min. afterburner residence time:

$$\text{AGAV} \le \text{V}/\text{ART}$$

33. Max. HCl content of afterburner exit gas:

$$100 \cdot \frac{\sum_{i=1}^{5} 2\text{Cl}_{mol}(i) + \sum_{j=9}^{11} 2\text{Cl}_{mol}(j)}{\text{AEXIT}} \le \text{AHCL}$$

34. Max. SO_2 content of afterburner exit gas:

$$100 \cdot \frac{\sum_{i=1}^{6} \text{S}_{mol}(i) + \sum_{j=9}^{12} \text{S}_{mol}(j)}{\text{AEXIT}} \le \text{ASO}_2$$

case, and therefore uncertainty must be incorporated into the optimization problem.

Uncertainties in waste characteristics may be mathematically quantified by the uncertain variables $\boldsymbol{u}$. In this work, the possible range of these uncertain variables will be assessed based on waste analysis data and some general knowledge of the analytical methods for measuring these characteristics. For example, waste heating value may be uncertain but its range is generally known within a 10% accuracy (Wilson, 1972). This range information leads to the definition of an uncertainty set U, in which the uncertainty variables $\boldsymbol{u}$ must lie. Several descriptions of the set U may be considered. The one used in this paper requires the least information about that set and presumes that the uncertainty variables are independent of one another and are bounded above and below. The resulting uncertainty description is:

$$U = \{\boldsymbol{u} = [u_1 u_2 \cdots u_q]^T \in R^q: u_k^l \le u_k \le u_k^u, \quad \forall\, k = 1, \cdots, q\} \tag{6}$$

where q is the number of uncertain variables.

Having described the range of waste characteristics, we now consider the mathematical quantification of the incinerator operability problem. The incinerator operates safely for a range of waste characteristics U if all the constraints associated with its operation are satisfied for all possible waste characteristics. Mathematically, if $f_i(\boldsymbol{x},\boldsymbol{u}) \le 0$, $\forall\, i = 1, \ldots, p$ denote operating constraints, which depend on both stream flow rates $\boldsymbol{x}$, and uncertainty variables $\boldsymbol{u}$, then safe incinerator operation results if, and only if,

$$f_i(\boldsymbol{x}, \boldsymbol{u}) \le 0 \quad \forall\, i = 1, \cdots, p \quad \forall\, \boldsymbol{u} \in U \tag{7}$$

or equivalently if, and only if,

$$h_i(x) \hat{=} \max_{\boldsymbol{u} \in U} f_i(\boldsymbol{x}, \boldsymbol{u}) \le 0 \quad \forall\, i = 1, \cdots, p \tag{8}$$

Having mathematically quantified safe incinerator operation, we can now proceed to the formulation of the optimal performance problem for safe incinerators. We demonstrate that this problem can be viewed as an embedded optimization problem.

The proposed optimization problem has a linear objective function, p nonlinear constraints, $f_i(\boldsymbol{x},\boldsymbol{u})$, which depend on both stream flow rates $\boldsymbol{x}$, and uncertainty variables $\boldsymbol{u}$, and m-p linear constraints, $g_j(\boldsymbol{x})$, that depend only on the stream flow rates $\boldsymbol{x}$. If we assume that there is no uncertainty, that is, that $\boldsymbol{u} = 0$, then the optimization problem becomes a linear programming problem that can be solved using the SIMPLEX algorithm. If $\boldsymbol{u}$ is nonzero, then the problem must be recast. The equivalence of Eqs. 7 and 8 suggests that these functional constraints may be replaced by the equivalent set of constraints, Eq. 8, whose evaluation involves the solution of a maximization problem. This in turn suggests that the problem under study is an embedded optimization problem that can be stated as follows:

$$\text{maximize } C^T\boldsymbol{x} \tag{9}$$

subject to

$$h_i(x) \hat{=} \max_{\boldsymbol{u} \in U} f_i(\boldsymbol{x}, \boldsymbol{u}) \le 0 \quad \forall\, i = 1, \cdots, p \tag{10}$$

$$g_j(\boldsymbol{x}) \le 0 \quad \forall\, j = p + 1, \cdots, m \tag{11}$$

where

$$\boldsymbol{u} = \{u = [u_1 u_2 \cdots u_q]^T \in R^q: u_k^l \le u_k \le u_k^u, \quad \forall\, k = 1, \cdots, q\} \tag{12}$$

The optimization problem, Eqs. 9–12, subordinates the optimization of the incinerator's economic performance to the satisfaction of all the operating and permit constraints for *all* allowable uncertainties $\boldsymbol{u}$ in the set U. That is, it subordinates economics to safety.

Solution Procedure

In general, Eqs. 9–12 can be solved by employing a cutting plane algorithm. This algorithm performs the optimization by solving a sequence of intermediate problems that employ a successively improving approximation of the feasible region. Detailed algorithm descriptions are given in optimization textbooks, for example, Reklaitis et al. (1983). Briefly, a rough approximation to the feasible region of the above problem can be made by choosing some of the linear constraints, such as Eq. 11. The optimum solution of this subproblem, $\boldsymbol{x}^{(1)}$, is then used to solve the problems $\max_u f_i(\boldsymbol{x}^{(1)},\boldsymbol{u})$ $\forall i = 1, \ldots, p$. If the optimum solutions of these problems violate a constraint, $f_i(\boldsymbol{x}^{(1)},\boldsymbol{u}^{(1)}) \le 0$ then this constraint, with the optimum value of $\boldsymbol{u} = \boldsymbol{u}^{(1)}$, is incorporated into the initial linear programming subproblem, thus adding a new constraint. The subproblem is then resolved and a new optimum solution, $\boldsymbol{x}^{(2)}$, will be obtained. This process terminates when no violation of $\max_u f_i(\boldsymbol{x}^{(n)},\boldsymbol{u}) \le 0$ takes place.

In this work, we do not use a cutting plane algorithm. Instead we capitalize on the linearity of $f_i(\boldsymbol{x},\boldsymbol{u})$ in $\boldsymbol{u}$ at a given $\boldsymbol{x}$, and demonstrate that the maximum of $f_i(\boldsymbol{x},\boldsymbol{u})$ for each $i = 1, \ldots, p$ occurs at some combination of uncertainty variables at their bounds, u_k^l and u_k^u. Proof of this argument is given in the appendix. Equations 9–12 can thus be solved by substituting for the u_ks all possible combinations of u_k^l and u_k^u, $\forall k = 1, \ldots, q$. Hence, p constraints of the form $f_i(\boldsymbol{x},\boldsymbol{u}) \le 0$ will be substituted by p^*2^q constraints, which are linear functions of $\boldsymbol{x}$, and the problem will be transformed to a linear programming problem. To limit the size of the problem, a combination of uncertain variable bounds (u_k^l and u_k^u) that most tightly restricts the feasible region can be chosen. This can be done by selecting the combinations of u_k^l and u_k^u that make the constraints $f_i(\boldsymbol{x},\boldsymbol{u})$ most positive.

This method of reducing the problem size can best be illustrated by an example. Take as an example, a ±10% uncertainty range in heating value measurements. The following constraints are impacted: KHIN (heat input to the kiln), CHIN (heat input to combined system of kiln and afterburner), KTEMP (kiln minimum temperature), and ATEMP (afterburner minimum temperature).

KHIN:

$$\sum_{i=1}^{5} (1 + u_i^\alpha)\text{HV}_i x_i + \text{HV}_6 x_6 \le \text{KHIN} \tag{13}$$

CHIN:

$$\sum_{i=1}^{5} (1 + u_i^\alpha)\text{HV}_i x_i + \sum_{j=9}^{11} (1 + u_j^\alpha)\text{HV}_j x_j + \text{HV}_6 x_6 + \text{HV}_{12} x_{12} \le \text{CHIN} \tag{14}$$

Table 2. Waste and Tank Analysis Data

	Streams							
	Low Heating Value (x_1, x_9)	Medium Heating Value (x_2, x_{10})	High Heating Value (x_3, x_{11})	Direct Burn Waste (x_4)	Sludge Waste (x_5)	Auxiliary Fuel (x_6, x_{12})	Steam (x_7, x_{13})	Water (x_8, x_{14})
c_i, objective func. coef.	1.0	1.0	1.0	1.0	1.0	−1.0	1.0	1.0
HV, kJ/kg	1,134	3,477	15,336	2,574	13,173	52,898	−2,778	−15,890
%C	1.96	6.50	30.69	4.40	24.62	76.10	0.0	0.0
%H	0.40	1.19	4.88	0.92	4.51	23.00	0.0	0.0
%O	0.13	1.22	6.13	0.29	4.60	0.00	0.0	0.0
%N	0.04	0.17	1.87	0.06	0.66	0.90	0.0	0.0
%Cl	0.44	4.35	42.15	0.26	16.49	0.00	0.0	0.0
%S	0.03	0.23	2.28	0.07	0.88	0.00	0.0	0.0
%H_2O	87.00	75.16	2.00	89.00	28.04	0.00	100.0	100.0
%Ash	10.00	11.18	10.00	5.00	20.20	0.00	0.0	0.0
ρ, kg/m^3	1,019	1,198	1,078	899	839	—	—	—

KTEMP:

$$\sum_{i=1}^{5} [-(1 + u_i^\alpha)\mathrm{HV}_i + \theta_i]x_i + \sum_{k=6}^{8} (-\mathrm{HV}_k + \theta_k)x_k \leq -\mathrm{KHLOS} \quad (15)$$

ATEMP:

$$\sum_{i=1}^{5} [-(1 + u_i^\alpha)\mathrm{HV}_i + \gamma_i]x_i + \sum_{k=6}^{8} (-\mathrm{HV}_k + \gamma_k)x_k + \sum_{j=9}^{11} [-(1 + u_j^\alpha)\mathrm{HV}_j + \gamma_j]x_j + \sum_{l=12}^{14} (-\mathrm{HV}_l + \gamma_l)x_l \leq -\mathrm{CHLOS} \quad (16)$$

where $u^\alpha = \pm 0.1$ are the limits of uncertainty in the heating values of the five waste streams; θ_n and γ_n are terms independent of heating values.

Uncertainty variables are associated with the heating values of the five waste types that are fed to the incinerator. No uncertainty is associated with fuel, steam, and water streams. Therefore, the number of possible combinations of the uncertainty variables bounds u^l and u^u is $2^5 = 32$. This means that the above four inequality constraints must be substituted by $4 \times 32 = 128$ constraints to account for the effect of uncertainty in heating values. Similarly, if uncertain variables appear in ten constraints, as in the case of uncertainty in halogen or nitrogen or water content, the number of constraints will be 10×32.

This problem can be solved using a SIMPLEX algorithm. However, by including uncertainties, the number of constraints and also the size of the problem increase significantly. The number of constraints can be reduced by taking another look at the mathematical form of the constraints. Examination reveals that for the KHIN and CHIN constraints, the combination $u_i^\alpha = u_i^u = 0.1, \forall i = 1, \ldots, 5$ makes the lefthand sides most positive and restricts the feasible region most tightly. For constraints KTEMP and ATEMP, the set $u_i^\alpha = u_i^l = -0.1, \forall i = 1, \ldots, 5$ is the most restrictive combination of uncertainty variables.

It must be emphasized that the proper set of u_i^α is not the same for all the active constraints. This is a direct consequence of the problem formulation. Indeed, since it is desired that all constraints be satisfied for all allowable uncertainties, it becomes clear that the worst uncertainty values for one constraint need

Table 3. Design-Related Data

	Stream													
Flow Rate kg/h	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Lower bound	0	113	227	0	181	0	0	0	301	0	0	0	0	0
Upper bound	408	1,089	694	980	1,837	1,837	3,400	3,400	612	1,089	980	1,837	3,400	3,400

Other Design Parameter

Parameter	Value	Parameter	Value
TVL	5.66 m^3	MXH	907.20 kg/h
TVM	5.66 m^3	AA	4.65 m^2
TVH	5.66 m^3	VL_T	68.00 m^3
TVDB	5.66 m^3	VM_T	68.00 m^3
TVSL	5.66 m^3	VH_T	68.00 m^3
MXL	907.20 kg/h	VDB_T	68.00 m^3
MXM	907.20 kg/h	VSL_T	68.00 m^3

Table 4. Permit and Operating Data

Parameter	Value
CWAST	3,384 kg/h
KASH	30 wt. %
AASH	15 wt. %
CASH	16 wt. %
CWATR	50 wt. %
CHLOR	30 wt. %
CSULF	5 wt. %
CNFUL	2 wt. %
KOXY	0.03 molar ratio
AOXY	0.03 molar ratio
LAGV	3.05 m/s
UAGV	4.57 m/s
ART	2 s
AHCL	2 mol %
ASO2	2 mol %
KHIN	7,937 kJ/h
CHIN	11,759 kJ/h
KHLOS	−397 kJ/h
AHLOS	−191 kJ/h
KTEMP	1,255 K
ATEMP	1,255 K
T_o	289 K
KEAP	0.5 molar ratio
AEAP	0.7 molar ratio
RTB	8 h
P	101.3 kPa
V	49.84 m^3
Steam: Saturated at 1,034 kPa	

not be the same for another. Thus, in each constraint the most limiting set of uncertain variable bounds, u_i^α, for the constraint is used. The solution obtained in this way insures feasibility for all the possible range of uncertainties. Thus for heating value uncertainties we substitute for Eqs. 13–16 the following:

KHIN:

$$\sum_{i=1}^{5} 1.1 \cdot \mathrm{HV}_i x_i + \mathrm{HV}_6 x_6 \leq \mathrm{KHIN} \quad (17)$$

CHIN:

$$\sum_{i=1}^{5} 1.1 \cdot \mathrm{HV}_i x_i + \sum_{j=9}^{11} 1.1 \cdot \mathrm{HV}_j x_j + \mathrm{HV}_6 x_6 + \mathrm{HV}_{12} x_{12} \leq \mathrm{CHIN} \quad (18)$$

KTEMP:

$$\sum_{i=1}^{5} (-0.9 \cdot \mathrm{HV}_i + \theta_i) x_i + \sum_{k=6}^{8} (-\mathrm{HV}_k + \theta_k) x_k \leq -\mathrm{KHLOS} \quad (19)$$

ATEMP:

$$\sum_{i=1}^{5} (-0.9 \cdot \mathrm{HV}_i + \gamma_i) x_i + \sum_{k=6}^{8} (-\mathrm{HV}_k + \gamma_k) x_k + \sum_{j=9}^{11} (-0.9 \cdot \mathrm{HV}_j + \gamma_j) x_j + \sum_{l=12}^{14} (-\mathrm{HV}_l + \gamma_l) x_l \leq -\mathrm{CHLOS} \quad (20)$$

Thus, after the proper selection of uncertainty variable bounds $\boldsymbol{u}^l$ and $\boldsymbol{u}^u$, the original problem becomes a linear programming problem with the same size as the problem formulated without uncertainty variables.

Case Studies

The optimization problem defined in Eqs. 9–12 was solved for the incineration configuration of Figure 1. The effect of uncertainties in heating values, waste moisture content, chlorine content, and nitrogen content were investigated. Specifically, the following five cases were examined:

1. No uncertainties in waste characteristics
2. ±50% uncertainty range in nitrogen content of the waste streams

Table 5. Optimum Solution to the Feed Blending Problem

No uncertainties in waste characteristics; flow rates in kg/h

Variable	Activity	Objective Function Coeff.	Lower Limit	Upper Limit
x_1	0.00	1.00	0.00	408.00
x_2	113.40	1.00	113.40	1,089.00
x_3	227.00	1.00	227.00	694.00
x_4	294.48	1.00	0.00	980.00
x_5	706.35	1.00	181.44	1,837.00
x_6	0.00	−1.00	0.00	1,837.00
x_7	809.74	0.00	0.00	3,400.00
x_8	0.00	0.00	0.00	3,400.00
x_9	612.00	1.00	301.18	612.00
x_{10}	793.79	1.00	0.00	1,089.00
x_{11}	636.98	1.00	0.00	980.00
x_{12}	144.21	−1.00	0.00	1,837.00
x_{13}	1,290.61	0.00	0.00	3,400.00
x_{14}	0.00	0.00	0.00	3,400.00

Constraint	Activity	Slack Activity	Lower Limit	Upper Limit
CWAST	3,384.00	0.00	None	3,384.00
MXL	612.35	294.85	None	907.20
MXM	907.20	0.00	None	907.20
MXH	863.66	43.60	None	907.20
TVL	4.81	57.49	None	62.30
TVM	6.06	56.24	None	62.30
TVH	6.40	55.90	None	62.30
TVDB	2.62	59.68	None	62.30
TVSL	6.74	55.56	None	62.30
KASH	−20,951.00	20,951.00	None	0.00
AASH	−9,277.90	9,277.90	None	0.00
CASH	−13,500.00	13,500.00	None	0.00
CWATR	0.00	0.00	None	0.00
CHLOR	−49,175.00	49,175.00	None	0.00
CSULF	−14,081.00	14,081.00	None	0.00
CNFUL	−4,490.00	4,490.00	None	0.00
KOXY	−6.20	6.20	None	0.00
AOXY	0.00	0.00	None	0.00
UAGV	778.24	6.72	None	784.96
LAGV	778.24	−254.94	523.30	None
ART	778.24	142.85	None	921.09
AHCL	0.00	0.00	None	0.00
ASO2	−1,385.60	1,385.60	None	0.00
KHIN	3,882.50	4,054.70	None	7,937.2
CHIN	9,691.40	2,067.40	None	11,758.8
KTEMP	−396.86	0.00	None	−396.86
ATEMP	−587.94	0.00	None	−587.94
OBJECTIVE	3,239.80	−3,239.80	None	None

3. ±10% uncertainty in heating value of the waste streams

4. ±10% uncertainty in chlorine (halogen) content of the waste streams

5. ±10% uncertainty in moisture content of the waste streams.

The data used for the case studies can be divided into three types; waste analysis, design and operating parameters, and permit data. The design and operating parameters and the permit data were obtained from an actual permit and the corresponding incinerator design. Waste stream data were selected based on the types of wastes expected to be fed to the incinerator, for example, halogenated organic solvents, aqueous wastes and sludges. The data are summarized in Tables 2–4.

Optimum solution of the problem formulated without uncertain variables, case 1, is given in Table 5, together with the values of some of the constraint equations using the optimum flow rates. These values are given in the column labeled Activity. The slack activity is defined as the amount by which the constraint can increase while maintaining feasibility, and indicates the sensitivity of solution to the constraint. The active constraints of the problem are CWAST, MXM, CWATR, AOXY, AHCL, KTEMP, and ATEMP; see Table 1 for descriptions of the constraints.

Table 6 summarizes the optimal flow rates and objective function values for cases 1–5. Values of the objective functions indicate that the sensitivity of the optimal solution to different uncertainties exhibits the following increasing order:

Nitrogen content uncertainties < heating value uncertainties < chlorine (halogen) content uncertainties < water content uncertainties

Comparison of optimum feed stream flow rates for the different cases reveals more details about the significance of the uncertainties on the optimum solution.

Halogen uncertainty has the most significant effect on the flow rates of streams that have a high chlorine content. As halogen content becomes uncertain, the flow rate of high heating value waste to the afterburner decreases to maintain the active constraint AHCL at the permitted level. At the same time, auxiliary fuel flow to the afterburner increases to maintain the minimum temperature, satisfying the constraint ATEMP. The flow rate of high heating value waste to the kiln remains at its minimum allowed value, while the flow rates of sludge waste and the low heating value waste increase to maximize throughput.

The impact of uncertainty in moisture content is different from that of uncertainty in halogen content in that the only sources for halogen are waste streams, while moisture is also introduced by steam and water streams. Thus, the heat release and the temperature constraints are more affected by variation in moisture content of the wastes.

Uncertainties in heating values result in an increase in the flow rate of sludge waste and a reduction in direct burn waste to the kiln. The steam flow rates to the kiln and afterburner decrease to maintain the active constraint CWATR at the permitted level. For the afterburner, the high heating value waste flow rate decreases such that the constraints AHCL and AOXY remain at their permitted limit. At the same time, the fuel flow rate increases to maintain minimum afterburner temperature.

The effect of nitrogen uncertainty on the optimal solution is less significant than the other uncertainties studied in this work. Indeed, a 10% uncertainty in nitrogen content resulted in no change in the optimal flow rates. A 50% uncertainty in nitrogen content results in a slight increase in sludge waste flow and some decrease in direct burn flow rate to the kiln, to satisfy the oxygen constraint AOXY and the temperature constraint KTEMP. The flow rate of high heating value waste to the afterburner shows some decrease while fuel flow rate increases and steam flow decreases slightly. As with all of the optimization calculations performed in this work, the sensitivity of the model to uncertainty in nitrogen loading will depend on the absolute nitrogen loading. Nonetheless, the modeling framework presented is capable of addressing these situations.

Table 6. Case Studies of Incinerator Optimization in the Presence of Uncertainty

Feed Type	Variable	Flow Rates,* kg/hr Case 1	Case 2	Case 3	Case 4	Case 5	Lower Limit	Upper Limit
Kiln								
Low heating value	x_1	0.00	62.91	0.00	201.21	0.00	0	408
Med heating value	x_2	113.40	113.40	113.40	113.40	113.40	113.4	1,089
High heating value	x_3	227.00	227.00	227.00	227.00	227.00	227	694
Direct Burn	x_4	294.48	231.86	272.53	0.00	0.00	0	980
Sludge	x_5	706.35	710.12	779.29	1,033.17	1,137.95	181	1,837
Fuel	x_6	0.00	0.00	0.00	0.00	0.00	0	1,837
Steam	x_7	809.74	793.59	580.51	0.00	0.00	0	3,400
Water	x_8	0.00	0.00	0.00	307.39	349.43	0	3,400
Afterburner								
Low heating value	x_9	612.00	612.00	612.00	612.00	539.77	301	612
Med heating value	x_{10}	793.79	793.79	793.79	793.79	793.79	0	1,089
High heating value	x_{11}	636.98	632.72	585.68	402.89	453.36	0	980
Fuel	x_{12}	144.21	149.09	162.44	249.48	246.02	0	1,837
Steam	x_{13}	1,290.61	1,288.36	1,080.54	293.99	0.00	0	3,400
Water	x_{14}	0.00	0.00	0.00	342.46	314.97	0	3,400
Objective function value, kg/h	—	3,239.8	3,234.7	3,221.3	3,134.0	3,019.3	—	—

Case 1: No uncertainty
Case 2: 50% uncertainty in nitrogen content
Case 3: 10% uncertainty in heating value
Case 4: 10% uncertainty in halogen content
Case 5: 10% uncertainty in moisture content

Acknowledgment

This work was supported by the National Science Foundation Engineering Research Center for Hazardous Substances Control, Grant No. CDR 8622184.

Notation

AA = cross-sectional area of afterburner, m^2
AASH = afterburner ash content, kg ash/100 kg waste feed
ACG = afterburner combustion gas flow rate, kmol/h
ACPEA = heat capacity of afterburner excess air times excess air flow rate, kJ/h
AEAP = excess air percentage for afterburner, mol excess air/mol stoich air
AEXIT = afterburner exit gas molar rate, kmol/h
AGAV = actual volumetric rate of combustion gas in the afterburner, m^3/s
AGV = combustion gas velocity in the afterburner, m/s
AHCL = HCl content of afterburner exit gas, mol HCl/100 mol exit gas
AHLOS = afterburner heat loss, kJ/h
AOXY = minimum oxygen percent of afterburner exit gas, mol O_2/mol exit gas
ART = residence time in afterburner, s
$(\%ash)_i$ = ash weight percent of stream *i*, kg ash/100 kg feed *i*
ASO2 = SO_2 content of afterburner exit gas, mol SO_2/100 mol exit gas
ATEMP = afterburner exit gas minimum temperature, K
C_g^* = average heat capacity of combustion gas *g* between the feed temperature and minimum operating temperature, kJ/kg · °C
C_i = profit function coefficients
C_P^{EA} = average heat capacity of excess air, kJ/kg · °C
CASH = incinerator ash content, kg ash/100 kg combined waste
CHIN = kiln + afterburner heat release, kJ/h
CHLOR = total chlorine content of feed streams, kg Cl/100 kg waste feed
$(\%Cl)_i$ = wt. % of Cl in stream *i*, kg Cl/100 kg feed *i*
$Cl_{mol}(i)$ = number of moles of Cl in stream *i*, kmol Cl in feed *i*/h
CNFUL = fuel nitrogen content of waste, kg N_2/100 kg waste feed
CSULF = total S content of feed streams, kg S/100 kg waste feed
CWAST = total waste feed rate to incinerator, kg/h
CWATR = total H_2O content of feed streams, kg H_2O/100 kg waste feed
EA = flow rate of excess air, kg/h
$f_i(x,u)$ = constraint *i*, a function of *x* and *u*
$h_i(x)$ = maximum value of the constraint *i* over the uncertainty set *U*
$(\%H_2O)_i$ = wt. % of water in stream *i*, kg H_2O/100 kg feed *i*
HV_i = heating value of stream *i*, kJ/kg
KASH = kiln ash content, kg ash/100 kg waste feed
KCG = Kiln combustion gas flow rate, kmol/h
KCPEA = heat capacity of kiln excess air times excess air flow rate, kJ/h
KEAP = excess air percentage for kiln, mol excess air/mol stoich air
KEXIT = kiln exit gas molar rate, kmol/h
KHIN = total kiln heat release, kJ/h
KHLOS = kiln heat loss, kJ/h
KOXY = minimum oxygen percent of kiln exit gas, mol O_2/mol exit gas
KTEMP = kiln exit gas minimum temperature, K
LAGV = lower limit for combustion gas velocity, m/s
LX_i = lower limit of flow rate of stream *i*, kg/h
MXH = maximum waste flow from high heating value tank, kg/h
MXL = maximum waste flow from low heating value tank, kg/h
MXM = maximum waste flow from medium heating value tank, kg/h
$(\%N)_i$ = wt. % of nitrogen in stream *i*, kg N/100 kg feed *i*
$(N_2)_{EA}$ = excess air nitrogen content, kmol N_2/h
$(O_2)_{EA}$ = excess air oxygen content, kmol O_2/h
$(O_2)_{stoich,A}$ = stoichiometric O_2 required for complete combustion in afterburner, kmol O_2/h
$(O_2)_{stoich,K}$ = stoichiometric O_2 required for complete combustion in kiln, kmol O_2/h
RTB = run time for a blend, h
$(\%S)_i$ = wt. % sulfur in *i*, kg S/100 kg feed *i*
$S_{mol}(i)$ = number of moles of S in stream *i*, kmol S in feed *i*/h
T_f = minimum operating temperature, K
T_o = reference or feed temperature, K
TVDB = minimum allowed waste volume in direct burn tank, m^3
TVH = minimum allowed waste volume in high heating value tank, m^3
TVL = minimum allowed waste volume in low heating value tank, m^3
TVM = minimum allowed waste volume in medium heating value tank, m^3
TVSL = minimum allowed waste volume in sludge tank, m^3
UAGV = upper limit for combustion gas velocity, m/s
UX_i = upper limit of flow rate of stream *i*, kg/h
u_i = uncertainty of waste property in stream *i*
u_i^l = lower bound on uncertainty variable *i*
u_i^u = upper bound on uncertainty variable *i*
V = volume of afterburner chamber, m^3
VDB_T = maximum waste volume in direct burn tank, m^3
VH_T = maximum waste volume in high heating value tank, m^3
VL_T = maximum waste volume in low heating value tank, m^3
VM_T = maximum waste volume in medium heating value tank, m^3
VSL_T = maximum waste volume in sludge, m^3 tank
x_i = flow rate of waste stream *i*, kg/h
ρ_i = density of stream *i*, kg/m^3
ν_{gi} = number of moles of combustion gas *g* generated per unit mass of feed *i*, kmol/kg feed *i*

Literature Cited

Behmanesh, N., Ph.D. Thesis, Univ. California, Los Angeles, (1990).
EPA: United States Environmental Protection Agency, Office of Solid Waste and Emergency Response, "The Waste System," U.S. Government Printing Office: 1989-6/9-248/00766.
Reklaitis, G. V., A. Ravindran, and K. M. Ragsdell, *Engineering Optimization, Methods and Applications*, Wiley, New York (1983).
Theodore, L., and J. Reynolds, *Introduction to Hazardous Waste Incineration*, Wiley, New York (1988).
Wilson, D.L., "Prediction of Heat of Combustion for Solid Wastes from Ultimate Analysis," *Environ. Sci. Tech.*, 6(13), 1119 (1972).

Appendix

At a given feasible point x, $f_i(x,u)$ are linear functions of u and $\max_u f_i(x,u)$ can be written as follows:

$$\max_{u \in U} \sum_k \alpha_k(x) \cdot u_k$$

subject to:

$$u_k^l \le u_k \le u_k^u \quad \forall\, k = 1, \cdots, q$$

The uncertainty variables, u_k, can be transformed to the following form

$$u_k = \beta_k u_k^l + (1 - \beta_k) u_k^u \quad \forall\, k = 1, \cdots, q$$

where $\quad 0 \le \beta_k \le 1$

then $$\max_{\beta_k \in [0,1]} \sum_k \alpha_k(x) \cdot [\beta_k u_k^l + (1 - \beta_k) u_k^u]$$

or $$\sum_k \alpha_k(x) \cdot u_k^u + \max_{\beta_k \in [0,1]} \sum_k \alpha_k(x) \beta_k (u_k^l - u_k^u)$$

Since $u_k^l < u_k^u$, β's must go to their bounds [0 if $\alpha_k(x)$ is positive, 1 if $\alpha_k(x)$ is negative] for the second term to become maximum. This implies that the maximum of $f_i(x,u)$ occurs only at a combination of u_k^l and u_k^u, $\forall k = 1, \ldots, q$.

Manuscript received Mar. 23, 1990, and revision received Sept. 12, 1990.